

Actuarial Challenge

Example taken from *The Drunkard's Walk, How Randomness Rules our Lives*
Leonard Mlodinow, Vintage Books

Actuaries are trained in mathematics, statistics and logic. The following challenge demonstrates the problem solving process that actuaries engage when doing forecasting. The TV Show *Let's Make a Deal* mimics this logic problem taken from the book *The Drunkards' Walk*.

Let's Make a Deal

Suppose the contestants on a game show are given the choice of three doors. Behind one door is a car; behind the others, goats. After a contestant picks a door, the host, who knows what's behind all the doors, opens one of the unchosen doors, which reveals a goat. He then says to the contestant, "Do you want to switch to the other unopened door?" Is it to the contestant's advantage to make the switch?

Answer

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Assume the car is behind door #1.

Scenario A

There's a $1/3$ chance that you'll choose door #1.

If you have #1, there's a $1/2$ chance that they'll show you door #2. If they do and you switch, you'll lose. Odds: $(1/3) * (1/2) = 1/6$ that Switch = Lose

If you have #1, there's a $1/2$ chance that they'll show you door #3. If they do and you switch, you'll lose. Odds: $(1/3) * (1/2) = 1/6$ that Switch = Lose

Scenario B

There's a $1/3$ chance that you'll choose door #2.

If you have #2, there's a 100% chance that they'll show you door #3. If you switch to #1 you'll win. Odds: $(1/3) * (1) = 1/3$ that Switch = Win

Scenario C

There's a $1/3$ chance that you'll choose door #3.

If you have #3, there's a 100% chance that they'll show you door #2. If you switch to #1 you'll win. Odds: $(1/3) * (1) = 1/3$ that Switch = Win

The odds that Switch = Lose from Scenario A are $1/6 + 1/6 = 2/6 = 1/3$

The odds that Switch = Win from Scenarios B and C are $1/3 + 1/3 = 2/3$